$\gamma3\pi$ and $\pi2\gamma$ form factors from dynamical constituent quarks

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Abstract

We study the form factors of the low energy anomalous $\pi2\gamma$ and $\gamma3\pi$ processes in the non-local chiral quark model which incorporates the momentum dependence of the dynamical quark mass and realizes correctly the chiral symmetries. The obtained slope parameter for $\pi2\gamma$ is in reasonable agreement with the direct experimental results but smaller than the ones invoking vector meson dominance. Our result for the $\gamma3\pi$ form factor interpolates between the two extremes of theoretical approaches, with the largest one provided by the vector meson dominance and the smallest one by the Schwinger-Dyson approach. But all of them are well below the single data point available so far. This situation will hopefully be clarified by the experiments at CEBAF and CERN.

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The $\pi^0\gamma\gamma$ ($\pi 2\gamma$) and $\gamma\pi^+\pi^0\pi^-$ ($\gamma 3\pi$) processes are the two simplest chiral anomaly-driven processes that involve electromagnetic interactions. A consideration of parity conservation, gauge invariance and Lorentz invariance implies the following structures for their amplitudes,

$$
\begin{aligned}
\mathcal{A}^{\pi 2\gamma}_{\mu\nu} &= \epsilon_{\mu\nu\rho\sigma} k_1^\rho k_2^\sigma A^{\pi 2\gamma}, \\
\mathcal{A}^{\gamma 3\pi}_{\mu} &= \epsilon_{\mu\nu\rho\sigma} p_+^\nu p_0^\rho p_-^\sigma A^{\gamma 3\pi}.
\end{aligned}
\tag{1}
$$

Here $k_{1,2}$ denote the outgoing momenta of the two photons with Lorentz indices μ and ν, and $p_{+,0,-}$ the incoming momenta of the three pions, for the two processes respectively. The dynamical information is encoded in the form factors $A^{\pi 2\gamma}$ and $A^{\gamma 3\pi}$ which are Lorentz invariant functions of the relevant momenta. In the low energy and chiral limit, they are completely determined by the chiral anomaly as summarized in the Wess-Zumino-Witten action to be [1]

$$
\begin{aligned}
A^{\pi 2\gamma}_0 &= \frac{e^2 N_c}{12\pi^2 f_\pi}, \\
A^{\gamma 3\pi}_0 &= \frac{e N_c}{12\pi^2 f_\pi^3},
\end{aligned}
\tag{2}
$$

where N_c and f_π are respectively the number of colors and the pion decay constant. Beyond the limit, their dependence on the relevant momenta is a reflection of the detailed strong dynamics. Since these processes involve only one or a few pions, they may provide an ideal testing ground for models of strong interactions.

The excellent agreement of $A^{\pi 2\gamma}_0$ with the experimental value extracted from the on-shell decay of $\pi^0 \to \gamma\gamma$ had historically constituted one of the first pieces of firm evidence that quarks carry three colors. When one of the photons is off-shell, the form factor can be parameterized by a slope parameter in the low energy region,

$$
A^{\pi 2\gamma}/A^{\pi 2\gamma}_0 = 1 + a\ x,
\tag{3}
$$

where $x = k^2/m_\pi^2$ describes the virtuality of the off-shell photon with momentum k. The slope parameter a has been measured both in the time-like region of k using the Dalitz decay $\pi^0 \to e^+ e^- \gamma$ and in the space-like region through the π^0 production in $e^+ e^-$ collisions. The direct results from TRIUMF and SINDRUM I in the first category are

respectively $a = 0.026 \pm 0.054$ [2] and $a = 0.025 \pm 0.014(\text{stat.}) \pm 0.026(\text{syst.})$ [3]. The CELLO group actually measured the form factor in the large space-like region and then extracted the slope parameter by extrapolation using the vector meson dominance to be $a = 0.0326 \pm 0.0026$ [4]. These results are consistent with each other within the quoted errors. Concerning the $\gamma 3\pi$ process the experimental situation is less clear. There has been so far one measurement [5] which seems to favor a larger value of $A_0^{\gamma 3\pi}$ than predicted by the chiral anomaly. Fortunately this situation will be much improved by the experiments at CEBAF [6] and CERN [7] which will measure the form factor $A^{\gamma 3\pi}$ in a wider range of kinematics. A more precise value of $A_0^{\gamma 3\pi}$ can then be extracted and the form factor will be available to distinguish the theoretical results based on hadronic models.

The low energy physics of the lowest-lying pseudoscalars may be described by a chiral Lagrangian which is a tower of terms in increasing order of energy expansion. The structures of terms at each order are completely determined by spontaneously broken chiral symmetries while their coefficients are left free. These parameters may be modelled by properly incorporating the relevant degrees of freedom in the intermediate-energy region. Of special interest in this regard are the quark-based models which may have a close connection to the underlying QCD dynamics. As is well-known, one feature of dynamical quarks is their running mass in the intermediate-energy region, which should have significant effects on low energy physics when the quarks are integrated out. This point has been nicely taken into account by Holdom and colloborators in their nonlocal constituent quark model [8][9][10]. Indeed, the coefficients in the $O(p^4)$ chiral Lagrangian for the lowest-lying pseudoscalars are expressed in terms of convergent integrals of the quark dynamical mass and their phenomenological values are well reproduced. The model has also been successful in modelling the low energy hadronic contributions to the running QED coupling at the Z boson pole [11], and in understanding the quark-hadron duality [12] and the electroweak couplings of constituent quarks themselves [13]. In this note we shall examine the other aspect of dynamical constituent quarks, namely, their implications on

the anomalous sector of the pseudoscalars, especially the form factors of the $\gamma 3\pi$ and $\pi 2\gamma$ processes . Since the Ward-Takahashi identities for flavor symmetries in QCD are built into the model of Holdom *et al.* we expect that the form factors so obtained should be comparable in quality to the coefficients in the $O(p^4)$ chiral Lagrangian derived from the model. Our results will be compared with those based on other approaches. The main feature here is that the effects of dynamical quark mass are included in a simplest possible form while at the same time avoiding introducing many free parameters.

The nonlocal constituent quark model is an effective theory in the intermediate energy regime. In this model all physics is assumed to be described by a chiral invariant action quadratic in quark fields. The dynamical quark mass $\Sigma(p)$ is incorporated into the action and its momentum-dependent nature leads to nonlocal interactions among dynamical quarks, Goldstone bosons and external gauge fields. Let us outline the action involving interactions with the external photon field $A_\mu(x)$ relevant to our discussion [10]. The interested reader should consult Refs. [8][9][10] for a complete account.

$$S \;=\; \int d^4x \; \bar\psi i\gamma_\mu D^\mu \psi - \int d^4x \int d^4y \; \Sigma(x-y)\bar\psi(x)\xi(x)X(x,y)\xi(y)\psi(y), \qquad (4)$$

where ψ represents the up and down quark fields with dynamical mass $\Sigma(p)$ whose Fourier transform is the quantity $\Sigma(x)$. And

$$
\begin{aligned}
D_\mu &= \partial_\mu - ieQA_\mu, \quad Q = \mathrm{diag}(2/3, -1/3),\\
\xi &= \exp(-i\pi\gamma_5/f_\pi), \quad \pi = \pi^a T^a,\\
X(x,y) &= P\exp(-i\int_x^y \Gamma_\mu(z)\, dz^\mu),\\
\Gamma_\mu &= i/2[\xi(\partial_\mu - ieQA_\mu)\xi^\dagger + \xi^\dagger(\partial_\mu - ieQA_\mu)\xi]\\
&= eQA_\mu + i/(2f_\pi^2)(\pi\partial_\mu\pi - \partial_\mu\pi\pi) + \cdots,
\end{aligned}
\qquad (5)
$$

where π^a is the pion field, T^a is the isospin matrix with $\mathrm{Tr}[T^a T^b] = \delta_{ab}/2$, and P stands for path-ordering. For convenience, we list in the following the relevant vertices appearing in our calculation of the $\gamma 3\pi$ and $\pi 2\gamma$ amplitudes. The QED vertex between quarks and the photon is modified to be

$$ieQ\left[\gamma_\mu - (p+p')_\mu R(p,p')\right], \qquad (6)$$

4

where $p(p')$ denotes the incoming (outgoing) momentum of the incoming (outgoing) quark line (same below), and

$$R(p, k) = \frac{\Sigma(p) - \Sigma(k)}{p^2 - k^2}.$$ (7)

We should mention in passing that the appearance of the R term in the QED vertex just fits the dynamical quark mass Σ appearing in the quark propagator so that the Ward identity still holds. The pion interaction with quarks is of a familiar form generalized from the constant mass case,

$$- f_\pi^{-1} \gamma_5 T^a [\Sigma(p) + \Sigma(p')].$$ (8)

The model generally contains nonlinear interactions of pions with quarks and photons due to the nonlinearly realized chiral symmetry and nonlocality. But we found that for the processes considered here only the following interaction involving two pions and two quarks can contribute at one loop level,

$$\frac{i}{2f_\pi^2} \left(\{T^a, T^b\} [\Sigma(p) + \Sigma(p + k_1) + \Sigma(p + k_2) + \Sigma(p')] \right.$$
$$\left. + \chi [T^a, T^b] [\Sigma(p + k_2) - \Sigma(p + k_1) + (k_1 - k_2) \cdot (p + p') R(p, p')] \right),$$ (9)

where the two pions carry the isospin indices a, b and the incoming momenta k_1, k_2 respectively. The parameter $\chi = 0$, 1 corresponds to the two versions [9][10] of the model. Since it makes little numerical difference, we shall henceforth take $\chi = 1$, corresponding to Ref. [10]. As one may easily figure out, only the χ term can contribute to Fig. 1b for the $\gamma 3\pi$ vertex while the first term in Eq. (9) cannot due to symmetry.

Let us consider the two processes whose Feynman diagrams are depicted in Fig. 1. Since we are interested in the form factors in the low energy region, we expand the amplitudes in the external momenta. The leading terms must be the same as predicted by the WZW action and thus universal to all models which correctly incorporate the chiral anomaly. In other words, they must be independent of the specific form of $\Sigma(p)$. This is indeed the case. For example the leading term in the $\pi 2\gamma$ amplitude is proportional to the following integral

$$\int_0^\infty dx \frac{d}{dx} \left(\frac{x}{x + \Sigma^2(\sqrt{x})} \right)^2$$

5

which is unity independently of Σ as long as Σ is finite in the Euclidean space. For the $\gamma 3\pi$ process the leading term is contributed only by Fig. 1a, whose integral can be simplified as

$$\int_0^\infty \left(-1 + \frac{x}{x + \Sigma^2(\sqrt{x})}\right) d \left(\frac{x}{x + \Sigma^2(\sqrt{x})}\right)^2$$

which is always $-1/3$ for a finite Σ in the Euclidean region. The subleading terms depend explicitly on the integrals of Σ which are collected using Mathematica. $A^{\pi 2\gamma}$ has been parameterized in Eqn. (3). For the $\gamma 3\pi$ process, as will become clear later on, we need to expand up to the $O(p^4)$ terms to display the kinematic variation of the form factor. Using the Bose symmetry we have

$$\frac{A^{\gamma 3\pi}}{A_0^{\gamma 3\pi}} = 1 + m_\pi^{-2} \sum_{i=1}^{2} b_i S_i + m_\pi^{-4} \sum_{i=1}^{6} c_i Q_i, \tag{10}$$

where S and Q are symmetrized Lorentz invariants of the momenta $p_{1,2,3} = p_{+,0,-}$,

$$\begin{aligned}
S_1 &= p_1^2 + p_2^2 + p_3^2, \\
S_2 &= p_1 \cdot p_2 + p_2 \cdot p_3 + p_3 \cdot p_1, \\
Q_1 &= (p_1^2)^2 + (p_2^2)^2 + (p_3^2)^2, \\
Q_2 &= p_1^2 p_2^2 + p_2^2 p_3^2 + p_3^2 p_1^2, \\
Q_3 &= p_1^2 p_1 \cdot (p_2 + p_3) + p_2^2 p_2 \cdot (p_3 + p_1) + p_3^2 p_3 \cdot (p_1 + p_2), \\
Q_4 &= p_1^2 p_2 \cdot p_3 + p_2^2 p_3 \cdot p_1 + p_3^2 p_1 \cdot p_2, \\
Q_5 &= p_1 \cdot p_2 p_2 \cdot p_3 + p_2 \cdot p_3 p_3 \cdot p_1 + p_3 \cdot p_1 p_1 \cdot p_2, \\
Q_6 &= (p_1 \cdot p_2)^2 + (p_2 \cdot p_3)^2 + (p_3 \cdot p_1)^2.
\end{aligned} \tag{11}$$

Note that the explicit factors of m_π are introduced for convenience although $m_\pi^{-2} b_i$ and $m_\pi^{-4} c_i$ actually do not depend on m_π.

The coefficients a, b_i and c_i are lengthy integrals involving the dynamical quark mass, which is in turn related to f_π by the Pagels-Stokar formula reproduced in the model [8][9][10],

$$f_\pi^2 = \frac{N_c}{4\pi^2} \int_0^\infty dx \frac{x(\Sigma - \frac{1}{2}x\Sigma')\Sigma}{(x + \Sigma^2)^2}, \tag{12}$$

with $\Sigma' = \frac{d}{dx}\Sigma$. A very simple parameterization for $\Sigma(p)$ in the Euclidean space was suggested by Holdom $et\ al.$, which incorporates the correct high energy behavior of the dynamical mass up to logarithms,

$$\Sigma(p) = \frac{(A+1)m^3}{p^2 + Am^2}, \tag{13}$$

6

where m is a typical mass scale of the constituent quark and is related to the parameter A through the Pagels-Stokar formula. Fixing $f_\pi = 84$ MeV in the chiral limit we therefore have only one free parameter. Since this simple ansatz is quite successful in reproducing phenomenological values of low energy quantities as mentioned previously, it will be used in our numerical analysis without further adaption.

Our results for the coefficients a, b_i and c_i are presented in Table 1 as a function of the parameter A in the same range of values as used previously, where the mass scale m is of order 300 MeV. Let us first discuss the slope parameter for the $\pi 2\gamma$ process. We get a stable result of $a = 0.02$ for the range of A in the table. This is in reasonable consistency with direct results from the Dalitz decays, but smaller than the one extracted from the large space-like region by extrapolation using vector meson dominance. The slope parameter has been studied in other approaches. The free quark loop [14] with a constant constituent mass m predicts $a = \frac{m_\pi^2}{12m^2}$, which is about 0.014 for $m = 330$ MeV. In the phenomenological approach of vector meson dominance the momentum dependence of the amplitude derives from the lowest-lying vector resonances and thus $a = \frac{m_\pi^2}{m_\rho^2} \sim 0.03$. Chiral perturbation theory is appropriate for dealing with low energy pion-photon interactions, but it is afflicted in the current case by the unknown counter-term parameters appearing in the $O(p^6)$ anomalous chiral Lagrangian. Assuming they are again saturated by vector mesons with a mean mass of $m_V^2 = (9m_\rho^2 + m_\omega^2 + 2m_\phi^2)/12$, the sum of loop and counter-term contributions gives $a = 0.032$ [15]. It is clear that our result is larger than the one in the constant quark mass model but smaller than the ones (both theoretical and experimental) invoking vector meson dominance.

For the $\gamma 3\pi$ process one has to examine the kinematic variation of the form factor to extract information on the coefficients b_i and c_i. In all of the three experiments available or approved, the photon and two of the pions, which we assume to be the first and second ones without loss of generality, are on-shell, $(p_1 + p_2 + p_3)^2 = 0$, $p_1^2 = p_2^2 = m_\pi^2$. (We take m_π to be the neutral pion mass below and ignore the small isopin breaking in mass.)

Table 1: Results of the coefficients a (in units of 10^{-2}), b_i (10^{-2}) and c_i (10^{-3}) as a function of the parameter A. The mass m (in units of MeV) is determined by the Pagels-Stokar formula. Ignoring the χ term in Eqn (9) would change b_i and c_i by less than 10%.

A	m	a	b_1	b_2	c_1	c_2	c_3	c_4	c_5	c_6
1	342	1.97	-2.29	-2.32	-1.02	-1.83	-2.23	-1.96	-2.52	-1.58
2	317	1.94	-2.37	-2.32	-0.97	-1.77	-1.95	-1.71	-1.89	-1.22
3	299	1.99	-2.52	-2.44	-1.09	-1.97	-2.13	-1.85	-1.97	-1.28
4	287	2.05	-2.67	-2.59	-1.27	-2.26	-2.45	-2.12	-2.25	-1.47
5	277	2.12	-2.82	-2.73	-1.47	-2.59	-2.83	-2.43	-2.61	-1.71

The experiment at Serpukhov and the one at CERN are of Primakoff type so that the third pion is also on-shell, $p_3^2 = m_\pi^2$, while the CEBAF experiment is to be done at a low momentum transfer of $p_3^2 \approx -m_\pi^2$. Defining the Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_2 + p_3)^2$ and $u = (p_3 + p_1)^2$, the form factor is a function of s and t with other kinematic variables completely fixed. It is then clear that there is no s or t dependence in the $O(p^2)$ terms of $A^{\gamma 3\pi}$ and this is the reason why we expand up to $O(p^4)$.

We plot in Fig. 2 our numerical results of the form factor $A^{\gamma 3\pi}/A_0^{\gamma 3\pi}$ at $A = 1$ as a function of s with fixed $t = -m_\pi^2$, for the Primakoff case (panel (a)) and the CEBAF case (panel (b)) respectively. Also shown are the results of other approaches, including the free quark loop with a constant constituent mass [16], the Schwinger-Dyson approach in the generalized impulse approximation [17], chiral perturbation theory with vector meson saturation [18], vector meson dominance [19] and its unitarized version [20]. The form factors expanded up to second order in s and t in the free quark loop and the Schwinger-Dyson approaches can be read off in the original papers. The chiral perturbation result augmented with vector meson saturation of counter-terms is [18]

$$
\begin{aligned}
\frac{A^{\gamma 3\pi}}{A_0^{\gamma 3\pi}} &= 1 + \frac{1}{2m_\rho^2}(s+t+u) + \frac{1}{32\pi^2 f_\pi^2}\left\{ -\frac{1}{3}(s+t+u)\ln\frac{m_\pi^2}{m_\rho^2} \right. \\
&\left. + \frac{5}{9}(s+t+u) + \frac{4m_\pi^2}{3}\left[f(m_\pi^2, s) + f(m_\pi^2, t) + f(m_\pi^2, u) \right] \right\},
\end{aligned}
\tag{14}
$$

where

$$f(m^2, q^2) = \begin{cases} (1-x)z \ln \dfrac{z+1}{z-1} - 2, & \text{for } x < 0, \\ (1-x)z \, 2 \arctan \dfrac{1}{z} - 2, & \text{for } 0 < x < 1, \\ (1-x)z \left[\ln \dfrac{1+z}{1-z} - i\pi \right] - 2, & \text{for } 1 < x, \end{cases}$$

$$z = \sqrt{\left| 1 - \frac{1}{x} \right|},$$

$$x = \frac{q^2}{4m^2},$$

which is different from the one quoted for the Primakoff case in Ref. [20]. The phenomenological approach of vector meson dominance gives [19]

$$\frac{A^{\gamma 3\pi}}{A_0^{\gamma 3\pi}} = -\frac{1}{2} \left[1 - \left(\frac{m_\rho^2}{m_\rho^2 - s} + \frac{m_\rho^2}{m_\rho^2 - t} + \frac{m_\rho^2}{m_\rho^2 - u} \right) \right], \tag{16}$$

which is unitarized to be [20]

$$\frac{A^{\gamma 3\pi}}{A_0^{\gamma 3\pi}} = -\frac{1}{2} \left[1 - \left(\frac{m_\rho^2}{m_\rho^2 - s} + \frac{m_\rho^2}{m_\rho^2 - t} + \frac{m_\rho^2}{m_\rho^2 - u} \right) \right] \frac{(m_\rho^2 - s)(m_\rho^2 - t)(m_\rho^2 - u)}{m_\rho^6 \, D_1(s) D_1(t) D_1(u)},$$

$$D_1(q^2) = 1 - \frac{q^2}{m_\rho^2} - \frac{q^2}{96\pi^2 f_\pi^2} \ln \frac{m_\rho^2}{m_\pi^2} - \frac{m_\pi^2}{24\pi^2 f_\pi^2} f(m_\pi^2, q^2).$$

$$\tag{17}$$

Note that the results for chiral perturbation and unitarized vector meson dominance are actually shown for $|A^{\gamma 3\pi} / A_0^{\gamma 3\pi}|$ since the form factor can become complex in these cases.

It is clear from Fig. 2 that the Schwinger-Dyson approach always gives the lowest values of the form factor while the vector meson dominance (especially its unitarized version) predicts the largest values and the steepest change in the kinematic region considered here. It is interesting to notice that in contrast to the case of the vertex $\pi 2\gamma$ the chiral perturbation theory predicts a much lower value of the $\gamma 3\pi$ amplitude than the vector meson dominance does. Our results interestingly interpolate the two extremes and are slightly larger than the one using a constant quark mass of 330 MeV.

We have studied the form factors of the low energy anomalous $\pi 2\gamma$ and $\gamma 3\pi$ processes in a simple quark-based model which incorporates the momentum dependence of the dynamical quark mass and realizes correctly the chiral symmetries. The obtained slope parameter for $\pi 2\gamma$ is in reasonable agreement with the direct experimental results from

TRIUMF and SINDRUM but smaller than the ones (both theoretical and experimental) invoking vector meson dominance. All theoretical predictions for the $\gamma 3\pi$ form factor are well below the single data point available so far. But there are also significant differences among these theoretical results. This situation will hopefully be clarified and distinguished by the experiments at CEBAF and CERN.

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Figure Captions

Fig. 1 Feynman diagrams for the vertices $\gamma 3\pi$ (a and b) and $\pi 2\gamma$ (c). The solid, dashed and wavy lines stand for the quark, pion and photon fields respectively.

Fig. 2 The form factor $A^{\gamma 3\pi}/A_0^{\gamma 3\pi}$ at $A = 1$ (solid curve) as a function of s/m_π^2 ($m_\pi = 135$ MeV) for the Primakoff case (panel (a)) and the CEBAF case (panel (b)) respectively. Also shown are the results of the following approaches: the free quark loop with a constant constituent quark mass of 330 MeV [16]; the Schwinger-Dyson approach [17]; chiral perturbation with vector meson saturation [18]; vector meson dominance [19] and its unitarization [20].
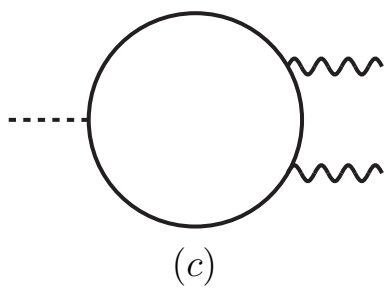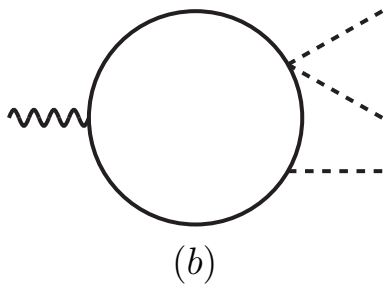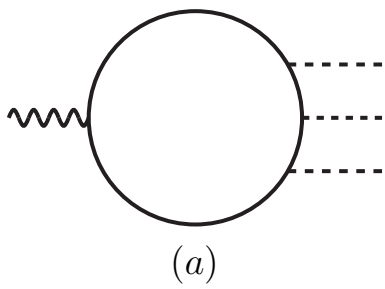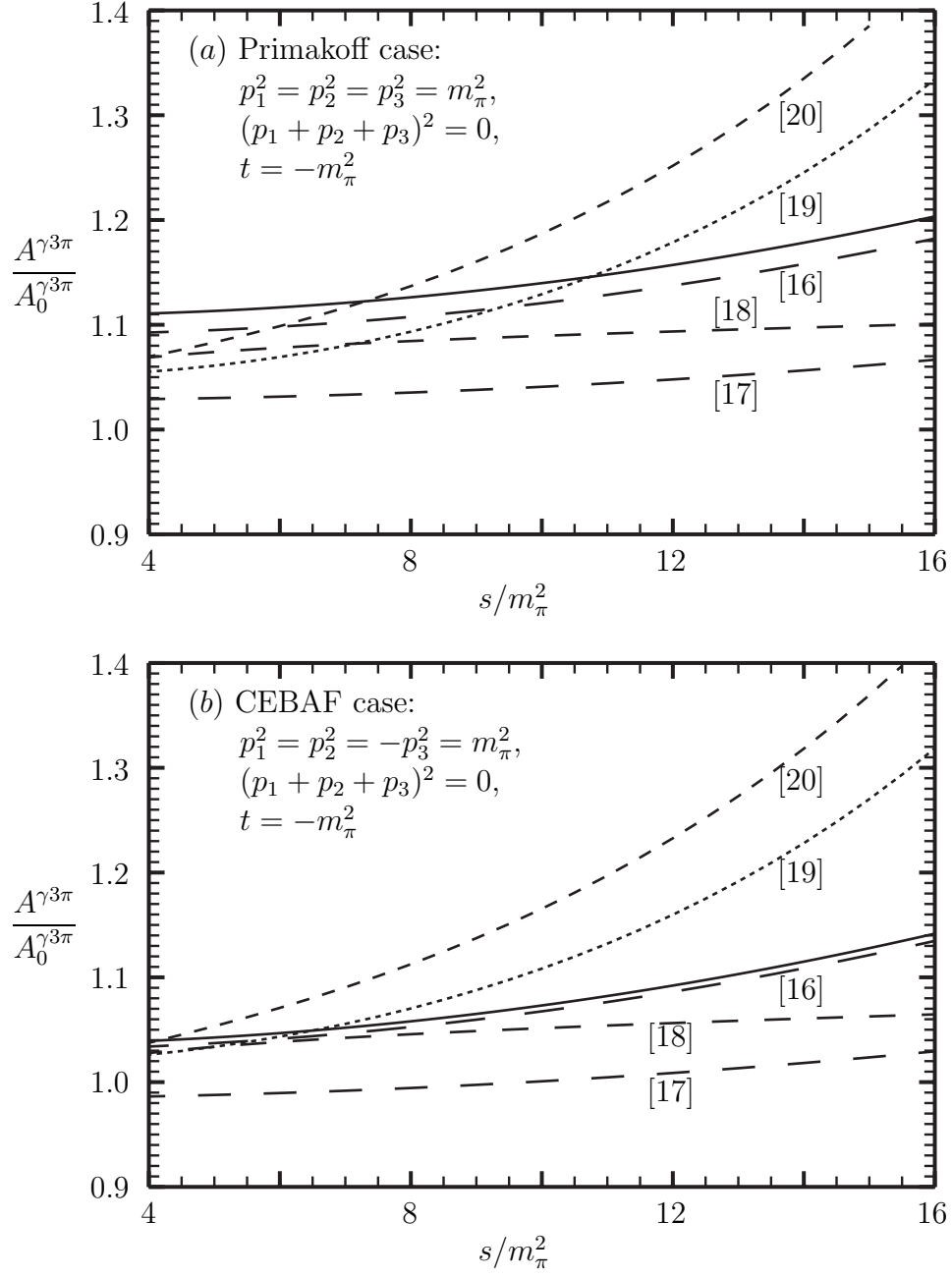
(a)

(b)

(c)

Figure 1

Figure 2